MATH AND SCIENCE @ WORK

AP* PHYSICS Educator Edition

TRAINING FOR A NEW SPACECRAFT
Moment of Inertia

Instructional Objectives

Students will

- derive an expression to determine the moment of inertia (MOI) of the Water Egress and Survival Trainer (WEST);
- relate the period of a torsional pendulum to rotational inertia; and
- explain the engineering behind the experimental set-up designed to measure the MOI of WEST.

Degree of Difficulty

For the average student in AP Physics, this problem is at a moderate difficulty level. An understanding of rotational inertia, parallel axis theorem, rotational mechanics, and simple harmonic motion is required.

Class Time Required

This problem requires 55–65 minutes.

- Introduction: 5–10 minutes
  - Read and discuss the background section with the class before students work on the problem. This background is identical to *Training for a New Spacecraft: Center of Gravity*.
- Student Work Time: 45 minutes
- Post Discussion: 5–10 minutes

Background

This problem is part of a series of problems that apply Math and Science @ Work in NASA's research facilities.

Orion is the most advanced spacecraft ever built and will carry up to four astronauts further into space than ever before. When paired with additional propulsion and life support systems, Orion can be reconfigured to take humans to asteroids or Mars.

Spacecraft shape must be considered when designing for the speed and heat of reentry encountered while returning from a deep-space mission. The laws of physics are no different now than in the twentieth century when NASA first designed for a mission to the Moon. For this reason, the shape

Grade Level
11–12

Key Topic
Moment of Inertia

Degree of Difficulty
Moderate

Teacher Prep Time
10 minutes

Class Time Required
55–65 minutes

Learning Objectives for AP Physics
Newtonian Mechanics:
-Circular motion and rotation
- Oscillations and gravitation

NSES Science Standards
- Physical Science
- Unifying Concepts and Processes
- Science and Technology

of the Orion spacecraft (Figure 1) is very similar to the Apollo Command Module (Figure 2), that took astronauts to the Moon in the 1960's and 70's.

Figure 1: The Orion Multi-Purpose Crew Vehicle being assembled and tested. Photo credit: Lockheed Martin

Figure 2: Apollo Crew Module

Then and now, astronauts must undergo a tremendous amount of training before traveling in a new spacecraft. Egress, or exiting the vehicle, is one activity that astronauts must practice and master in order to have a safe return from space. Although the capsule concept is not new, many physical parameters are quite different than ones used in previous vehicles. These differences require new simulators to be engineered so astronauts can train effectively.

The Water Egress and Survival Trainer (WEST) is a new simulator that will replicate the geometry and mass properties of the Orion flight capsule and will be used exclusively for egress training. Once WEST has been constructed and egress procedures have been developed, astronaut training will be conducted in NASA's Neutral Buoyancy Laboratory (NBL) located in Houston, Texas.

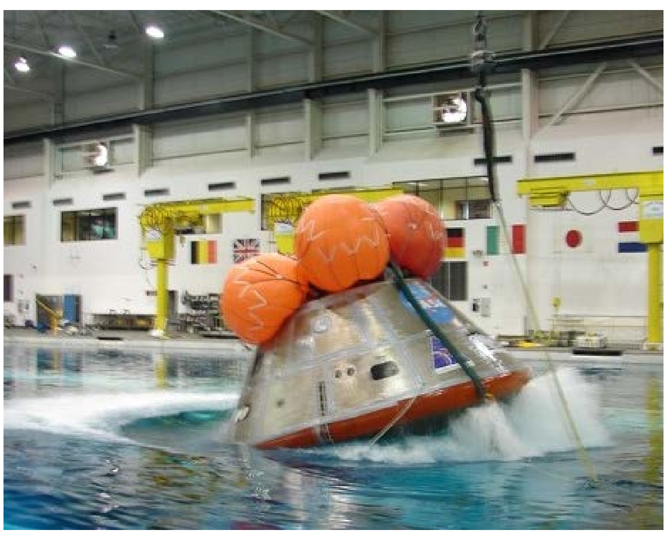

Figure 3: Crew Module Uprighting System Test being performed in the NBL with a full-scale Orion test article

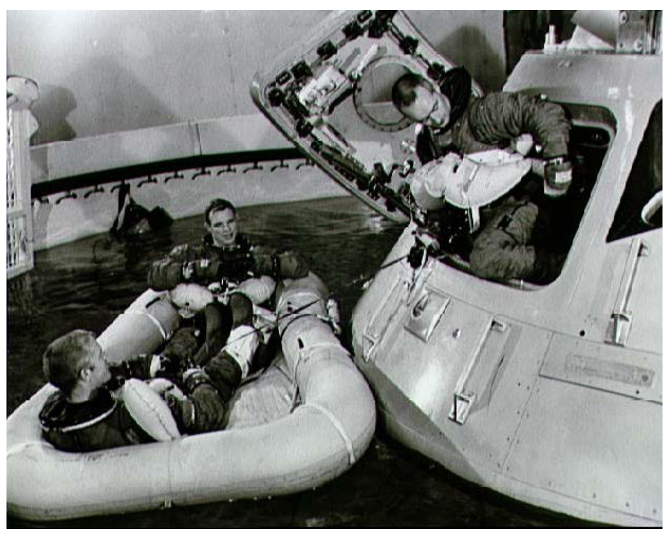

Figure 4: Apollo 9 astronauts training with a mockup of the Apollo crew capsule in a pool

Like Apollo, Orion will return to Earth for an ocean water landing, but egress training methods between these missions will be drastically different. Apollo training was performed before the NBL was constructed. Training was conducted statically in pools (Figure 4) and at sea with unpredictable sea states and weather. In contrast, WEST will be attached to hydraulic actuators on the pool floor of the NBL to emulate variable sea states. This capability will provide economic, scheduling, and training benefits—great advantages for astronaut training in preparation for Orion missions.

Learning Objectives for AP Physics

Newtonian Mechanics

- Circular motion and rotation
  - Rotational kinematics and dynamics
- Oscillations and gravitation
  - Simple harmonic motion – dynamics
  - Pendulum and other oscillations

NSES Science Standards

Physical Science

- Motions and forces

Unifying Concepts and Processes

- Change, constancy, and measurement
- Form and function

Science and Technology

- Abilities of technological design

Problem and Solution Key (One Approach)

WEST is essentially a hollow cone. Inside, it will contain four astronauts, life support systems, propellant, control systems, and other flight and on-orbit equipment. Its combined mass, and the location of its mass, will affect the stability of the capsule.

One describing property of stability is the moment of inertia (MOI). MOI describes the rotational properties around any axis, including the capsules ability to right itself after an ocean landing (Figure 5). Understanding the MOI will allow training engineers to add, remove, or relocate weight to facilitate different capsule orientations and egress scenarios, including rough seas that could result in a capsized craft.

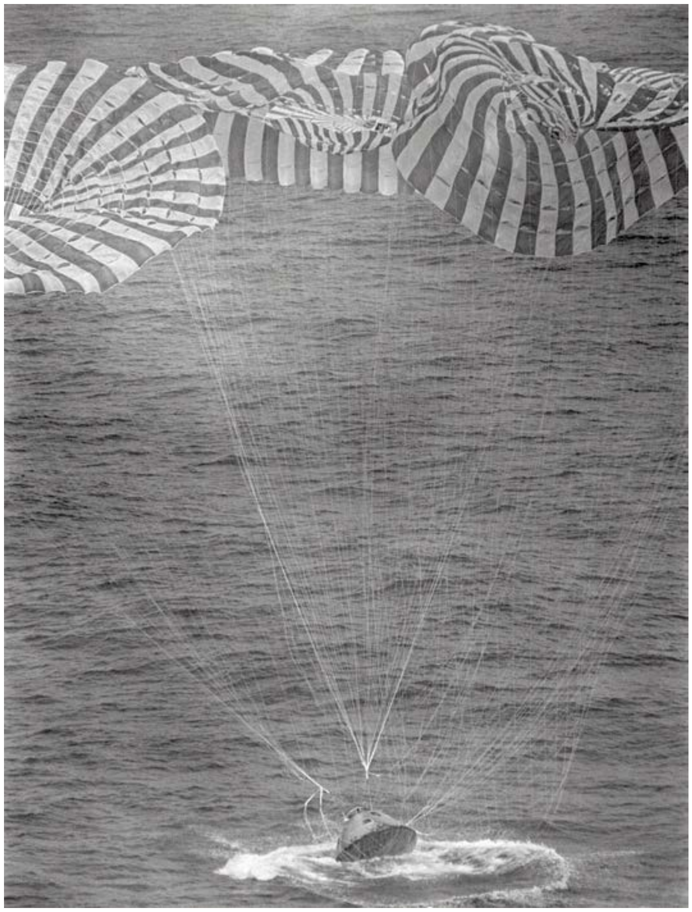

Figure 5: The Apollo 9 Command Module splashes down in the Atlantic recovery area, concluding a successful 10-day, Earth orbital mission.

Identifying the MOI of WEST requires engineering a device that will allow WEST to experience rotational motion. The most efficient design is to use a torsion rod to relate the MOI to simple harmonic motion.

An experimental design using a ¼ scale model of Orion will first be built to understand the physical properties before constructing the full-scale WEST simulator. Figure 6 shows one experimental design set-up providing all necessary equipment components that would enable identification of the MOI for this scaled model.

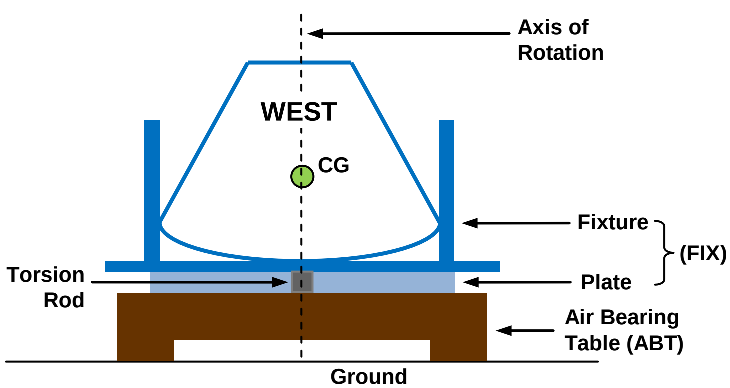

Figure 6: WEST experimental design set-up

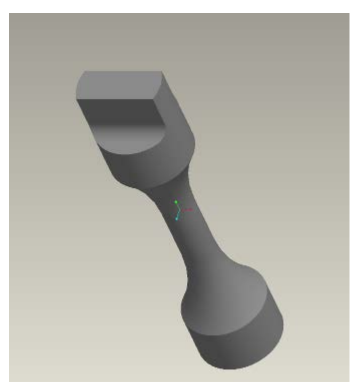

Figure 7: Torsion rod

The torsion rod (Figure 7) would be one component. It would be made of aluminum and have an equivalent stiffness that could be simplified to a torsion spring constant, κ (kappa). The circular end would be mounted to an air-bearing table. WEST would sit in a fixture that would allow the orientation to be either vertical (as shown in Figure 6) or horizontal. WEST and the fixture would then be set and fixed onto a plate supported by the air-bearing table. This plate would have an opening allowing the torsion rod to extend into the base of the fixture. The air-bearing table will be able to support the fixed system (WEST, fixture, and plate) by blasting air upward, similar to a heavy duty air hockey table.

By rotationally displacing the fixed system by 1°, the body would oscillate in a frictionless manner on the torsion rod about the axis of rotation noted in Figure 6. This movement can be described by simple harmonic motion. The experimental design set-up illustrated in Figure 6 is a torsional pendulum. As such, the period of oscillation can be related to the MOI of WEST.

Below, question A asks for a generalization of the determination of the MOI of WEST and question B asks for a mathematical determination of the MOI of WEST.

A. Use the experimental design set-up of WEST (Figure 6) to determine an equation describing the MOI of WEST.

I. Construct a general expression describing the total rotational inertia of the system (I_{TOT}) in terms of the rotational inertias of the different components of the experimental design set-up.

$$I_{TOT} = I_{WEST} + I_{FIX} + I_{ABT}$$

II. The parallel axis theorem acknowledges the possibility that rotation will occur around an axis other than an axis through the center of gravity. Accounting for this possibility yields the following equation, where *I* is rotational inertia, *m* is mass, and *r* is the distance from the axis of rotation.

$$\left(I_{TOT} + m_{TOT}r_{TOT}^2\right) = \left(I_{WEST} + m_{WEST}r_{WEST}^2\right) + \left(I_{FIX} + m_{FIX}r_{FIX}^2\right) + \left(I_{ABT} + m_{ABT}r_{ABT}^2\right)$$

From an engineering stand point, several of the terms can be eliminated because they are either negligible or zero. Carefully and critically analyze the equation with Figure 6 and propose which terms can be eliminated with a justification for each.

$m_{TOT}r_{TOT}^2$: *The center of gravity of the entire system is in-line with the central axis of rotation, so it is zero.*

I_{ABT} : *The MOI of the air-bearing table is quite negligible and will contribute little, if any, rotational inertia to the entire system.*

$m_{ABT}r_{ABT}^2$: *The air-bearing table's central axis is the point of reference, meaning* r_{ABT} *is zero, and therefore this element is zero.*

III. Derive an equation that describes the MOI for WEST, I_{WEST}.

$$I_{WEST} = (I_{TOT} - I_{FIX}) - m_{WEST}r_{WEST}^2 - m_{FIX}r_{FIX}^2$$

B. The fact that the MOI set-up behaves as a torsional pendulum allows for the rotational inertia to be related to the period of oscillation. The air-bearing table, having outputs, will be connected to a computer allowing physical measurements to be collected by data acquisition. The period of oscillation and how far the torsion rod is actually displaced rotationally will be measured.

I. Show that the acceleration of a torsional pendulum is given by $\alpha = -\frac{\kappa}{I}\theta$.

$$\tau = -\kappa\theta$$
$$-\kappa\theta = I\alpha$$
$$\alpha = -\frac{\kappa}{I}\theta$$

II. The acceleration of a harmonic oscillator is defined as $a = \frac{d^2x}{dt^2}$ *or* $a = -\omega^2 x$ (where $x = A\cos(\omega t + \phi)$). Show that the angular velocity of the WEST oscillator is defined by $\omega = \sqrt{\frac{\kappa}{I}}$.

$\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$ *is of the same form as* $\frac{d^2x}{dt^2} = -\omega^2 x$*; therefore,*

$\omega^2 = \frac{\kappa}{I}$ *and* $\omega = \sqrt{\frac{\kappa}{I}}$.

III. Recalling that the period of rotational motion is defined as $T = \frac{2\pi}{\omega}$, show that the rotational inertia of a harmonic oscillator is given by $I = \left(\frac{\kappa}{4\pi^2}\right)T^2$.

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{\kappa}}$, simplifying to $T^2 = \frac{4\pi^2}{\kappa}I$, and resulting in $I = \left(\frac{\kappa}{4\pi^2}\right)T^2$

IV. Derive an equation for the MOI of WEST in terms of *T*, *m*, *r*, and physical constants.

$$I_{WEST} = \left(\frac{\kappa}{4\pi^2}\right)\left(T_{TOT}^2 - T_{FIX}^2\right) - m_{FIX}r_{FIX}^2 - m_{WEST}r_{WEST}^2$$

Scoring Guide

Suggested 15 points total to be given.

Question		Distribution of points
A	*7 points*	2 points for part I: 2 point for I_{WEST}, I_{FIX}, I_{ABT} (1 point awarded if only 2 correct inertial components are present) 3 points for part II: 1 point for each correct term with explanation 2 points for part III: 1 point for an equation consistent with responses in question A part II 1 point for a correct equation
B	*8 points*	2 points for part I: 1 point for $\tau = -\kappa\theta$ 1 point for setting $-\kappa\theta = I\alpha$ 2 points for part II: 1 point for recognizing the two forms of acceleration 1 point for setting $\omega^2 = \frac{\kappa}{I}$ 1 point for part III for correct substitution of ω from question B part II 3 points for part IV: 2 points for an equation consistent with response in question A part III 1 point for a correct equation

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school AP Physics instructors.

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